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INFLUENCE OF VOLTAGE HARMONICS ON DISTRIBUTION TRANSFORMER LOSSES

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Abstract - ***This paper proposes a fast calculation method to determine the transformer losses. A methodology will be explained to incorporate the loss calculation in a harmonic load-flow program. The transformer model is obtained by coupling the electrical equations with a magnetic equivalent circuit, which allows to compute the currents in the primary and secondary windings next to the magnetic field and the magnetic flux density on the basis of the applied voltages. The magnetic field is applied in a static Preisach model to quantify the hysteresis losses, and the flux density is considered to calculate the instantaneous power loss. Simulations have been performed to determine the influence of voltage harmonic distortion on the transformer losses.***

Introduction

For a power system, the distribution system loss has become more and more of a concern because of the growth of load demand and the wide area it covers. In this frame harmonic losses are a topic of great interest [1, 2]. In the analysis of distribution network losses, the additional losses in transformers, due to the harmonic voltages, can be significant [3]. The harmonic voltages cause significant higher eddy current losses in transformers through the fact that these losses are proportional with the square of the frequency. Because of these higher losses, the working temperature can be higher than expected resulting in a lower life span of the transformer. A practical solutions for this life span problem is the overdimension of the transformer as has been documented in the IEC. 61378.1 standard. An overdimensioned transformer is simply larger so it can carry of the warmth resulting from the extra losses caused by the harmonics. An other solution is the use of K-rated transformers, they have a restricted eddy current loss.

The objective of this research is to calculate the losses of a transformer in a harmonic load-flow program. Several methodologies have already been presented in literature [4–6]. In these references finite element analysis has been applied to determine the transformer losses. The method applied in this paper has been developed in order to minimize the computing time and to incorporate the transformer model in a harmonic load flow program. In order to minimize the computing time the losses are determined in a post calculation process. This approximation is acceptable as has been mentioned in [3].

The applied transformer model determines the currents in the primary and secondary windings next to the magnetic field and the magnetic flux density on the basis of the given voltages. The transformer model is obtained by coupling the electrical equations with a magnetic equivalent circuit of a transformer in symmetrical components. This type of space discretization yields the benefit that the losses can be calculated in the legs and the yokes separately. As compared to [3] unbalances between the phases and the coupling between phases can be taken into account. The transformer model is based on the harmonic balance method, which gives the advantage that the transformer model can be easily implemented in a harmonic load-flow program. Given the frequency components of the voltages at the primary and secondary side of the transformer the magnetic flux density and the magnetic field can be calculated. The magnetic fields are used in a static Preisach model to perform a post processing in order to determine the hysteresis losses. The magnetic flux density is used to calculate the eddy current losses.

The transformer model and the post processing method are then applied in a case study to determine the influence of the harmonic voltages on the distribution transformer losses.

Transformer Loss Calculation

In order to quantify the transformer losses the magnetic field and the magnetic flux density have to be determined given the frequency components of the voltages at the primary and secondary side. These quantities are then used in a post processing methodology to determine the hysteresis losses and the eddy current losses.

Nonlinear Harmonic Transformer Model

The magnetic equivalent circuit consists of magnetomotive force (mmf) sources, which are determined by the currents, and reluctances, with the fluxes Φ as variables. The mmf drop $\mathcal{F}$ across a reluctance $\mathcal{R}$ is given by:

$$\mathcal{F} = \mathcal{R}(\Phi)\,\Phi \tag{1}$$

The nonlinear magnetic behaviour, i.e. $\mathcal{R}(\Phi)$, will be approached by means of an anhysteretic magnetization curve.

As the MEC is considered in the frequency domain, the reluctances $\mathcal{R}$ have to be resolved in frequency components in order to establish the harmonic balance of (1). Due to the nonlinear relationship $\mathcal{R}(\Phi)$, the computation of $\mathcal{R}$ is performed in the time domain. Therefore, the flux Φ is evaluated in a discrete number of points in time by an inverse FFT of its frequency components, and the corresponding reluctance $\mathcal{R}$ is computed. Finally, the spectrum of $\mathcal{R}$ is computed by means of an FFT.

When considering (1), the multiplication of a reluctance, with space harmonic order n and time harmonic order k, and a flux, with space harmonic order m and time harmonic order l, leads to a mmf with time harmonic order h and space harmonic order κ. A first contribution is

$$\begin{aligned} h &= k + l \\ \kappa &= \mathrm{m} + \mathrm{n} + 3\mathrm{q}, \end{aligned}$$

and the second one is

$$\begin{aligned} h &= k - l \\ \kappa &= \mathrm{m} - \mathrm{n} + 3\mathrm{q}, \end{aligned}$$

if $k > l$ and else

$$\begin{aligned} h &= l - k \\ \kappa &= \mathrm{n} - \mathrm{m} + 3\mathrm{q}. \end{aligned}$$

In the above equations, q has to be -1, 0 or 1 in order to obtain a value for κ that equals 0, 1 or 2, for the homopolar, the direct and the inverse component respectively.

The electric and magnetic transformer equations of a wye G-wey G three-phase three-legged transformer can be derived from Fig. 1, which represent the electric and magnetic equivalent circuit of the transformer. The flux paths of the direct and inverse components of the leg fluxes are assumed to close through the yokes such that only the path of the homopolar leg fluxes close (necessarily) via the air. This assumption is justified by the fact that the yokes – even if saturated – have a much lower reluctance than the flux path through the air, resulting in the grey closing path for the homopolar component with reluctance $3\mathcal{R}_{\mathrm{d}}$ (Fig. 1), where $\mathcal{R}_{\mathrm{d}}$ is the constant reluctance of the air branch.

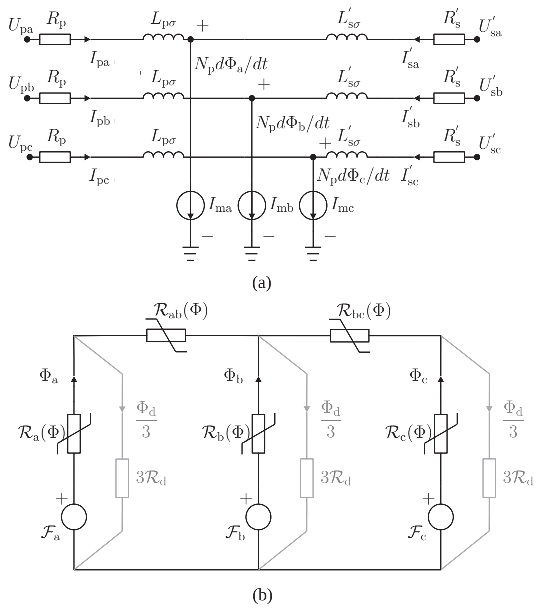

Fig. 1 Equivalent electric (a) and magnetic circuit (b) of a three-phase three-legged transformer.

The electrical equations, for the three phases $(\mathrm{a}, \mathrm{b}, \mathrm{c})$, derived from Fig. 1(a) are

$$\begin{aligned} U_{\mathrm{pn}} &= R_{\mathrm{p}} I_{\mathrm{pn}} + L_{\mathrm{p}\sigma} \frac{dI_{\mathrm{pn}}}{dt} + N_{\mathrm{p}} \frac{d\Phi_{\mathrm{n}}}{dt} \\ U'_{\mathrm{sn}} &= R'_{\mathrm{s}} I'_{\mathrm{sn}} + L'_{\mathrm{s}\sigma} \frac{dI'_{\mathrm{sn}}}{dt} + N_{\mathrm{p}} \frac{d\Phi_{\mathrm{n}}}{dt} \quad (\mathrm{n} = \mathrm{a}, \mathrm{b}, \mathrm{c}) \end{aligned} \tag{2}$$

where

- $I_{\mathrm{pn}}, I'_{\mathrm{sn}}, U_{\mathrm{pn}}, U'_{\mathrm{sn}}$: the currents and the voltages of the primary and the secondary windings referred to the primary side
- Φ_{n}: the magnetic flux in the legs
- $R_{\mathrm{p}}, R'_{\mathrm{s}}, L_{\mathrm{p}\sigma}, L'_{\mathrm{s}\sigma}$: the winding resistances and the constant leakage inductances referred to the primary side
- $N_{\mathrm{p}}, N_{\mathrm{s}}$: the number of turns

The relations between the magnetic flux and the currents obtained from this circuit are:

$$\begin{aligned} 0 &= N_{\mathrm{p}}(I_{\mathrm{pa}} + I'_{\mathrm{sa}}) - N_{\mathrm{p}}(I_{\mathrm{pb}} + I'_{\mathrm{sb}}) \\ &\quad - (\mathcal{R}_{\mathrm{a}} + \frac{2}{3}\mathcal{R}_{\mathrm{ab}})\Phi_{\mathrm{a}} + (\mathcal{R}_{\mathrm{b}} + \frac{1}{3}\mathcal{R}_{\mathrm{ab}})\Phi_{\mathrm{b}} + \frac{\mathcal{R}_{\mathrm{ab}}}{3}\Phi_{\mathrm{c}} \\ 0 &= N_{\mathrm{p}}(I_{\mathrm{pc}} + I'_{\mathrm{sc}}) - N_{\mathrm{p}}(I_{\mathrm{pb}} + I'_{\mathrm{sb}}) \\ &\quad - (\mathcal{R}_{\mathrm{c}} + \frac{2}{3}\mathcal{R}_{\mathrm{bc}})\Phi_{\mathrm{c}} + (\mathcal{R}_{\mathrm{b}} + \frac{1}{3}\mathcal{R}_{\mathrm{bc}})\Phi_{\mathrm{b}} + \frac{\mathcal{R}_{\mathrm{bc}}}{3}\Phi_{\mathrm{a}} \\ 0 &= \sum_{\mathrm{n}=\mathrm{a,b,c}} N_{\mathrm{p}}(I_{\mathrm{pn}} + I'_{\mathrm{sn}}) - \sum_{\mathrm{n}=\mathrm{a,b,c}} (\mathcal{R}_{\mathrm{n}} + 3\mathcal{R}_{\mathrm{d}})\Phi_{\mathrm{n}}, \end{aligned} \tag{3}$$

where $\mathcal{R}_{\mathrm{n}}$ is the nonlinear reluctance of leg n, $\mathcal{R}_{\mathrm{ab}}$ and $\mathcal{R}_{\mathrm{bc}}$ are the nonlinear reluctances of the yoke between two legs.

Table 1 The input data for the 400kVA distribution transformer

Winding resistances and leakage inductances
$R_\mathrm{p} = 4.1106\ \Omega \quad R_\mathrm{s} = 2.157\ \mathrm{m}\Omega$ $L_\mathrm{p} = 0.25528\ \mathrm{mH} \quad L_\mathrm{s} = 0.1362\ \mu\mathrm{H}$
Core dimensions
$A = 21940\ \mathrm{mm}^2 \quad l_\mathrm{leg} = 460\ \mathrm{mm} \quad l_\mathrm{yoke} = 303\ \mathrm{mm}$
Material properties
$\sigma = 2.38e6\ \mathrm{S/m} \quad d = 0.30\ \mathrm{mm}$
Primary and secondary windings and voltages
$N_\mathrm{p} = 1212 \quad V_\mathrm{p} = 10\ \mathrm{kV}$ $N_\mathrm{s} = 28 \quad V_\mathrm{p} = 230\ \mathrm{V}$

Depending on the time harmonic order of the reluctance and the flux, these equations (3) can be rewritten in symmetrical components of the reluctances $\underline{\underline{\mathcal{R}}}_\mathrm{a}$, $\underline{\underline{\mathcal{R}}}_\mathrm{b}$ and $\underline{\underline{\mathcal{R}}}_\mathrm{c}$.

The equations in symmetrical components are of the form

$$G(x) = S(x)x - b = 0 \tag{4}$$

The components of the state vector x are the harmonic currents and the flux. The vector b consists of the frequency components of the voltages at the primary and secondary side. The Newton-Raphson method is used to solve 4, which implies the use of the iterative application of the algorithm:

$$x^{m+1} = x^m - DG(x^m)^{-1}G(x^m) \tag{5}$$

until convergence is reached. The calculation of the Jacobian $DG(x)$ can be obtained numerically as $DG(x) = S + (\partial S/\partial x) \cdot x$ due to the particular form of the nonlinear equation $G(x) = S(x)x - b = 0$. So the flux in the legs and the yokes are determined and so the magnetic flux density can be found. When the anhysteretic magnetization curve is considered the magnetic field in the legs and the yokes is obtained.

Post Processing Methodology

The transformer model uses an anhysteretic magnetization curve depicted from tests with Epstein strips. To determine the hysteresis losses the Preisach model is applied. The Preisach model uses a large number of measured hysteresis loops with varying amplitude to compose an Everett-map of the material. The Preisach model determines for a given step in the magnetic field, resulting from the transformer model, the concording step in the magnetic flux density by interpolation in the Everett-map, taken into account the history of the material [7]. The surface of the received hysteresis loop determines the hysteresis losses.

The eddy current losses per unit volume is give by [8]:

$$P_{cl} = \frac{\sigma d^2}{12}\omega^2 B^2 \tag{6}$$

where σ represents the conductivity of the material and d the thickness of one sheet. The magnetic flux density used to obtain the eddy current losses is depicted out of the transformer model.

Case Study

The properties of the 400kVA distribution transformer under test are given in Table 1. The distribution transformer is connected in delta star configurations which implies that the equations derived above have to altered. For a nongrounded wye the equation that gives the relation between the zero sequence current and voltage can be withdrawn. For the connection in delta , the zero sequence component of the voltage has to be set to zero. The anhysteretic magnetization curve derived from the material is shown in Fig. 2.

In order to study the influence of the harmonic voltage distortion on the distribution transformer losses several cases have been considered. The simulation results are given in Table 2. In the first column the applied voltages

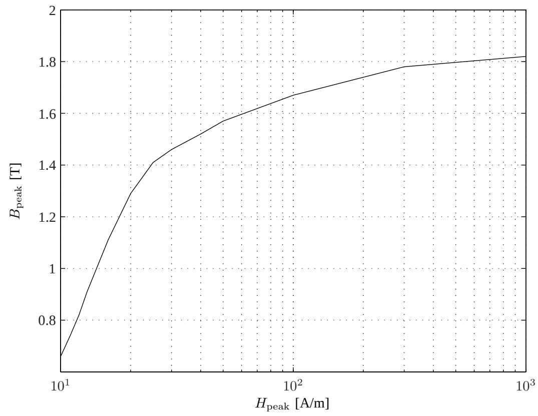

Fig. 2   The anhysteretic magnetization curve.

Table 2   Harmonic voltages and iron losses

case	v_p, v_s (pu)	P_{hyst} (kW/m^3)	P_{cl_1} (kW/m^3)	P_{cl_h} (W/m^3)
1	1^{th} 1	12.695	12.627	0.000129
2	1^{th} 1 7^{th} 0.03	12.983	12.628	1.140
3	1^{th} 1 7^{th} 0.05	13.157	12.628	3.163
4	1^{th} 1 7^{th} $0.05\angle 90°$	12.773	12.628	3.144
5	1^{th} 1 7^{th} $0.05\angle 180°$	12.138	12.627	3.153
6	1^{th} 1 7^{th} $0.05\angle 270°$	12.788	12.627	3.173
7	1^{th} 1 5^{th} 0.05	11.922	12.627	3.142
8	1^{th} 1 5^{th} $0.05\angle 180°$	13.334	12.628	3.177
9	1^{th} 1 5^{th} $0.05\angle 180°$ 7^{th} 0.05	13.832	12.629	6.345

at the primary and secondary side are given. A seventh harmonic is a direct component where the fifth harmonic is an inverse component. The second column represents the hysteresis losses (kW/m^3). In the third and the forth column the fundamental (kW/m^3) and the harmonic (W/m^3) eddy current losses are given, respectively. In fig. 3 the hysteresis loops of case 1 and case 9 are depicted.

When the simulations results are examined, the following conclusions can be drawn. First, the influence of the harmonic voltages on the hysteresis losses is dependent on the voltage waveforms. It can be stated that the surface under the voltage waveform is the determining factor to quantify the hysteresis losses. In such a way, one can remark that the peak value is not the soul important parameter (case 5 yields the highest peak value) but also the trapezoidal shape of the voltage waveform is important. The losses increase about 9% between case 1 and case 9, the cause for this rather low percentage can be explained when fig. 2 and 3 are considered. The steep slope explains the low difference in losses when the magnetic flux density changes. In fig. 3 one can see that when the transformer material is saturated the hysteresis loops have a minimum of extra surface for a higher magnetic flux density, so the influence of peak values is smaller. A second conclusion that can be drawn

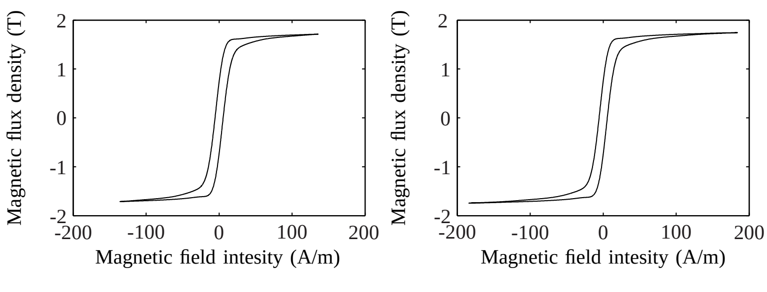

Fig. 3   Hysteresis loop of case 1 (left) and case 9 (right).

from the simulation results is that the fundamental eddy current losses are influenced, though minimalistic, by the harmonic voltages. This results from the coupling between the different harmonics in the transformer model. The harmonic eddy current losses are in the cases considered maximum 0.5% of the fundamental eddy current losses. So it there can be concluded that the harmonic voltages have little influence on the global eddy current losses.

Conclusion

The transformer model and the post processing methodology give a good insight in the iron losses in a transformer. The origin of these extra losses has been discussed. In this discussion there is observed that the material properties have a big influence on the extra losses caused by the harmonics and so an economical assessment can be made between the overdimension of the transformer or the investment in better materials to expand the life span of the transformer.

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